Anomalous Quantum Diffusion in Order-Disorder Separated Double Quantum Ring

Santanu K. Maiti[†,‡,*]

[†] *Theoretical Condensed Matter Physics Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata-700 064, India*
[‡] *Department of Physics, Narasinha Dutt College, 129, Belilious Road, Howrah-711 101, India*

Abstract

A novel feature for control of carrier mobility is explored in an order-disorder separated double quantum ring, where the two rings thread different magnetic fluxes. Here we use simple tight-binding formulation to describe the system. In our model, the two rings are connected through a single bond and one of the rings is subjected to impurity, keeping the other ring as impurity free. In the strong impurity regime, the electron diffusion length increases with the increase of the impurity strength, while it decreases in the weak impurity regime. This phenomenon is completely opposite to that of a conventional disordered double quantum ring, where the electron diffusion length always decreases with the increase of the disorder strength.

PACS No.: 73.23.Ra; 73.23.-b; 73.63.-b

Keywords: Persistent current; Drude weight; Double quantum ring; Impurity.

[*] **Corresponding Author**: Santanu K. Maiti
Electronic mail: santanu.maiti@saha.ac.in

1 Introduction

Over the last few decades, the physics at sub-micron length scale provides enormous evaluation both in terms of our understanding of basic physics as well as in terms of the development of revolutionary technologies. In this length scale, the so-called mesoscopic or nanoscopic regime, several characteristic quantum length scales for the electrons such as system size and phase coherence length or elastic mean free path and phase coherence length are comparable. Due to the dominance of the quantum effects in the mesoscopic/nanoscopic regime, intense research in this field has revolved its richness. The most significant issue is probably the persistent currents in small normal metal rings. In thermodynamic equilibrium, a small metallic ring threaded by magnetic flux ϕ supports a current that does not decay dissipatively even at non-zero temperature. It is the well-known phenomenon of persistent current in mesoscopic normal metal rings which is a purely quantum mechanical effect and gives an obvious demonstration of the Aharonov-Bohm effect.[1] The possibility of persistent current was predicted in the very early days of quantum mechanics by Hund,[2] but their experimental evidences came much later only after realization of the mesoscopic systems. In 1983, Büttiker *et al.*[3] predicted theoretically that persistent current can exist in mesoscopic normal metal rings threaded by a magnetic flux ϕ, even in the presence of impurity. In a pioneering experiment, Levy *et al.*[4] first gave the experimental evidence of persistent current in the mesoscopic normal metal ring, and later, the existence of the persistent current was further confirmed by several experiments.[5−8] Though the phenomenon of persistent current has been addressed quite extensively over the last twenty years both theoretically[9−27] as well as experimentally,[4−8] but yet we cannot resolve the controversy between the theory and experiment. The main controversies come in the determinations of (a) the current amplitude, (b) flux-quantum periodicities, (c) low-field magnetic susceptibilities, etc. In recent works,[24−26] we have pointed out that the higher order hopping integrals, in addition to the nearest-neighbor hopping integral, have a significant role to enhance the current amplitude (even an order of magnitude). In other recent work,[27] we have focused that the low-field magnetic susceptibility can be predicted exactly only for the one-channel systems with fixed number of electrons, while for all other cases it becomes random. To grasp the experimental behavior of the persistent current, one has to focus attention on the interplay of quantum phase coherence, disorder and electron-electron correlation and this is a highly complex problem.

Using the advanced molecular beam epitaxial growth technique, one can easily fabricate a quantum system where the impurities are located only in some particular region of the system, keeping the other region free from any impurity. This is completely opposite from a conventional disordered system, where the disorders are given uniformly throughout the system. Traditional wisdom is that, the larger the disorder stronger the localization.[28] However, some recent experimental studies[29−31] as well as theoretical investigations[32−35] on these special class of systems where the disorders are not distributed uniformly, have yielded completely different behavior which predicts that the electron diffusion length decreases in the weak disorder regime, while it increases in the strong disorder regime. Motivated with these results, in this article, we focus our attention in an order-disorder separated double quantum ring system. To reveal the variation of the electron diffusion length in such a particular system, here we study the behavior of persistent current and Drude weight and our results may illuminate some of the unusual experimental results for such diverse transport property. The parameter Drude weight D characterizes the conducting nature of the system as originally introduced by Kohn.[36] In our present model, two mesoscopic rings, threaded by different magnetic fluxes, are connected by a single bond and impurities are given in any one of these two rings, while the other ring becomes impurity free. For this order-disorder separated double quantum ring, we observe an anomalous behavior of electron mobility in which the electron diffusion length increases with the increase of the impurity strength in the strong impurity regime, while the diffusion length decreases in the weak impurity regime. This phenomenon is completely opposite to that of a conventional disordered double quantum ring, in which the electron diffusion length always decreases with the increase of the disorder strength.

In what follows, we describe the model and the method in Section 2. Section 3 contains the significant results and the discussion, and finally, we summarize our results in Section 4.

2 The model and the method

The schematic representation of a double quantum ring is shown in Fig. 1 where the two rings, threaded by different magnetic fluxes, are connected by a single bond. In the non-interacting picture, the system

is usually modeled by a single-band tight-binding Hamiltonian,

$$\begin{aligned} H &= \sum_i \epsilon_i^I c_i^\dagger c_i + v_I \sum_{<ij>} \left[e^{i\theta_I} c_i^\dagger c_j + e^{-i\theta_I} c_j^\dagger c_i \right] \\ &+ \sum_k \epsilon_k^{II} c_k^\dagger c_k + v_{II} \sum_{<kl>} \left[e^{i\theta_{II}} c_k^\dagger c_l + e^{-i\theta_{II}} c_l^\dagger c_k \right] \\ &+ v_{\alpha\beta} \left[c_\alpha^\dagger c_\beta + c_\beta^\dagger c_\alpha \right] \end{aligned} \quad (1)$$

Here ϵ_i^I's (ϵ_i^{II}'s) are the site energies in the ring I (ring II), $c_i^\dagger$ ($c_k^\dagger$) is the creation operator of an electron at site i (k) of the ring I (ring II) and c_i (c_k) is the annihilation operator of an electron at site i (k) of the ring I (ring II), v_I (v_{II}) is the

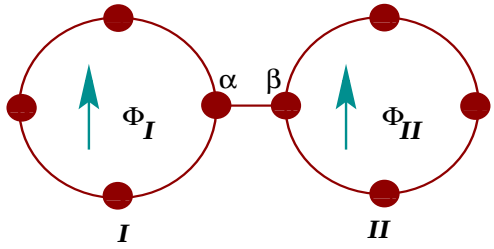

Figure 1: Schematic view of a double quantum ring in which the two rings thread magnetic fluxes ϕ_I and ϕ_{II} respectively. These two rings are connected through the lattice sites α and β. The filled circles correspond to the position of the atomic sites (for color illustration, see the web version).

hopping strength between nearest-neighbor sites in the ring I (ring II), and $v_{\alpha\beta}$ gives the hopping strength between these two rings. In this expression, $\theta_I = 2\pi\phi_I/N_I$ and $\theta_{II} = 2\pi\phi_{II}/N_{II}$ are the phase factors due to the fluxes ϕ_I and ϕ_{II} (measured in units of $\phi_0 = ch/e$, the elementary flux quantum), respectively where N_I and N_{II} correspond to the total number of atomic sites in the ring I and ring II, respectively. In order to introduce the impurities in the system, we choose the site energies (ϵ_i's, omitting the ring index in the superscript) from the relation: $\epsilon_i = W\cos(i\lambda\pi)$, where W is the strength of the disorder and λ is an irrational number, and as a typical example we take it as the golden mean $\left(1+\sqrt{5}\right)/2$. Setting $\lambda = 0$, we get back the pure system with identical site potential W. The idea of considering such an incommensurate potential is that, for such a correlated disorder we do not require any configuration averaging and therefore the numerical calculations can be done in the low cost of time. Now to achieve the order-disorder separated double quantum ring, we introduce the correlated disorder in any one of the rings, keeping the other one as impurity free.

At absolute zero temperature, the persistent currents in the two rings can be calculated from the expressions,

$$I(\phi_I) = -\frac{\partial E(\phi_I, \phi_{II})}{\partial \phi_I} \quad (2)$$

$$I(\phi_{II}) = -\frac{\partial E(\phi_I, \phi_{II})}{\partial \phi_{II}} \quad (3)$$

where, $I(\phi_I)$ and $I(\phi_{II})$ correspond to the currents in the ring I and ring II, respectively and $E(\phi_I, \phi_{II})$ represents the ground state energy of the complete system. We evaluate this energy exactly to understand unambiguously the anomalous behavior of persistent current, and this is achieved by exact diagonalization of the tight-binding Hamiltonian Eq. (1).

Now the response of the double quantum ring system to a uniform time-dependent electric field can be determined in terms of the Drude weight D,[37–38] a closely related parameter that characterizes the conducting nature of the system as originally noted by Kohn.[36] The Drude weights for the two rings can be calculated through the relations,[39]

$$D_I = \frac{N_I}{4\pi^2} \left(\frac{\partial^2 E(\phi_I, \phi_{II})}{\partial {\phi_I}^2} \right) \Bigg|_{\phi_I \to 0, \phi_{II} \to 0} \quad (4)$$

$$D_{II} = \frac{N_{II}}{4\pi^2} \left(\frac{\partial^2 E(\phi_I, \phi_{II})}{\partial {\phi_{II}}^2} \right) \Bigg|_{\phi_I \to 0, \phi_{II} \to 0} \quad (5)$$

where D_I and D_{II} represent the Drude weights for the ring I and ring II respectively.

Our main aim in this article is the determination of the conducting properties of an order-disorder separated double quantum ring, which can be computed through the parameters D_I and D_{II}. From these parameters we can clearly describe the mobility of the charge carriers in the system and accordingly, the variation of the electron diffusion length might be expected.

3 Results and discussion

In the order-disorder separated double quantum ring, we introduce the correlated disorder in ring I (for the sake of simplicity), keeping the ring II as impurity free. Throughout the numerical computations, we take the values of the different parameters as: $v = -1$, $v_{\alpha\beta} = -1$ and for the sake of simplicity, we use the units where $c = 1$, $e = 1$ and $h = 1$. During these calculations, we fix the chemical potential (μ) for all the systems to a constant value 0. The main focus of this article is to describe how the disordered states affect the ordered states in the

order-disorder separated system. Since for such a system the impurities are introduced in the ring I, we evaluate the conducting properties of the double quantum ring by measuring the Drude weight D_{II}. This actually provides the response of the ordered states in presence of the disordered states. Otherwise, if we measure the parameter D_I, then

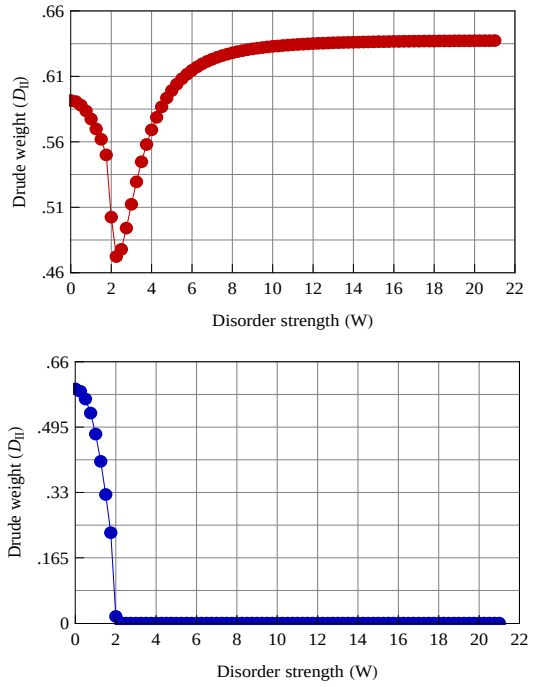

Figure 2: Drude weight (D_{II}) of the ring II as a function of the disorder strength (W) for the systems with $N_I = 30$, $N_{II} = 30$ and the fixed chemical potential $\mu = 0$. The red and the blue lines correspond to the order-disorder separated and the complete disordered double quantum ring systems respectively (for color illustration, see the web version).

we will get the trivial result as obtained in a traditional disordered system since the response of the disordered states will not be changed by coupling these states with the ordered states.

In Fig. 2, we show the variation of the Drude weight D_{II} as a function of the disorder strength W for some double quantum rings, where we choose $N_I = 30$ and $N_{II} = 30$. The chemical potential μ is fixed to 0. The red and the blue curves represent the results for the order-disorder separated and the complete disordered double quantum rings, respectively. From the results it is observed that, in the complete disordered double quantum ring the Drude weight sharply decreases with the increase of the disorder strength and eventually it drops to zero. Therefore, we can say that for such a system the electron diffusion length as well as the electron mobility decreases sharply with the disorder strength. Such a behavior can be well understood from the theory of Anderson localization, where we get more localization with the increase of the disorder strength.[28] The anomalous behavior is observed when the impurities are given only in any one of the two rings, keeping the other one as impurity free i.e., for the order-disorder separated system. Our results predict that the Drude weight initially decreases with the increase of the disorder strength, but after reaching to a minimum it again increases with the strength of the disorder. Such a phenomenon is completely opposite to that of the traditional disordered system and can be justified in the following way. For the order-disorder separated double quantum ring, the energy spectra of the disordered ring are gradually separated from the energy spectra of the ordered ring with the increase of the disorder strength W. Therefore, the influence of random scattering in the ordered ring due to the strong localization in the disordered ring decreases. It has been examined that the energy spectrum of the order-disorder separated double quantum ring with large disorder contains localized tail states with much small and central states with much large values of localization length, contributed approximately by disordered and ordered rings, respectively. Hence the central states gradually separated from the tail states and delocalized with the increase of the strength of the disorder. Thus we see that, for the coupled order-disorder separated double quantum ring, the coupling between the localized states with the extended states is strongly influenced by the strength of the disorder, and this coupling is inversely proportional to the disorder strength W. Accordingly, in the limit of weak disorder the coupling effect is significantly high, while the coupling effect becomes very weak in the strong disorder regime. Hence, in the limit of weak disorder the electron transport is strongly influenced by the impurities at the disordered ring such that the electron states are scattered more and therefore the electron diffusion length decreases which manifests the lesser electron mobility. On the other hand, for the stronger disorder limit the extended states are weakly influenced by the disordered ring and the coupling effect gradually decreases with the increase of the disorder strength which provides the larger electron mobility in the strong disorder limit. This reveals that the electron diffusion length increases in this limit. For large enough impurity strength, the extended states are almost unaffected by the im-

purities at the disordered ring and in that case the electrons are carried only by these extended states in the ordered ring which is the trivial limit. So the novel phenomenon will be observed only in the intermediate limit of W.

In order to emphasize the dependence of the electron mobility on the system size, here we focus our attention on the results those are plotted in Fig. 3. In this figure, we display the Drude weight for some typical double quantum rings, where we fix $N_I = 50$

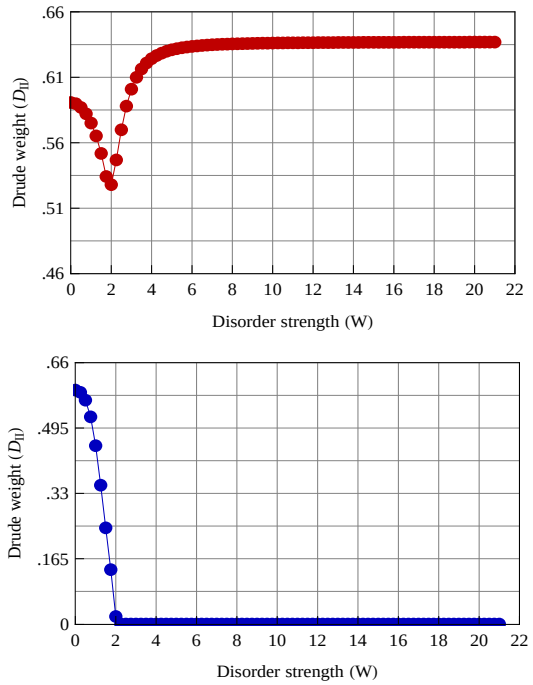

Figure 3: Drude weight (D_{II}) of the ring II as a function of the disorder strength (W) for the systems with $N_I = 50$, $N_{II} = 50$ and the chemical potential $\mu = 0$. The red and the blue lines correspond to the order-disorder separated and the complete disordered double quantum ring systems respectively (for color illustration, see the web version).

and $N_{II} = 50$. Similar to the previous systems, here we also take $\mu = 0$ for these systems. The red and the blue lines correspond to the same meaning as in Fig. 2. From this figure (Fig. 3) it is also observed that, the Drude weight in the order-disorder separated double quantum ring decreases with the increase of the disorder strength W in the weak disorder regime, while it increases with the strength W in the strong disorder regime. On the other hand, the Drude weight always decreases with the strength of the disorder for the complete disordered system, as expected. Though the results plotted in Fig. 3 seem to be quite similar in nature with the results those are described in Fig. 2, but the significant point is that, the typical magnitude of the Drude weight strongly depends on the size of both these two rings which manifest the finite quantum size effects. Now the other significant factor that raises to our mind is the existence of the location of the minimum in the Drude weight versus disorder curves of the order-disorder separated double quantum rings. This minimum can be implemented as follows. The carrier mobility in the order-disorder separated double quantum ring is controlled by the two competing mechanisms. One is the random scattering in the ordered ring due to the localization in the disordered ring which tends to decrease the carrier mobility, and the other one is the vanishing influence of random scattering in the ordered ring due to the strong localization in the disordered ring which provides the enhancement of the carrier mobility. Depending on the ratio of the total number of atomic sites in the disordered ring to the total number of atomic sites in the ordered ring, the vanishing effect of random scattering from the ordered states dominates over the non-vanishing effect of random scattering from these states for a particular disorder strength ($W = W_c$), which provides the location of the minimum in the Drude weight versus disorder curve.

4 Concluding remarks

In conclusion, we have established a novel feature for control of the electron diffusion length in an order-disorder separated double quantum ring in which the two rings thread different magnetic fluxes. From our study it has been observed that, in the order-disorder separated double quantum ring, the electron diffusion length increases with the increase of the disorder strength in the strong disorder regime, while it decreases in the weak disorder regime. Such a peculiar behavior is completely opposite to that of the conventional disordered systems, where the electron diffusion length always decreases with the increase of the disorder strength. Lastly, we have noticed that, both the electron mobility and the location of the minimum in the Drude weight versus disorder curve strongly depend on the size of both the two rings which manifest the finite quantum size effects. Our theoretical results in this article might be helpful to illuminate some of the unusual experimental phenomena which have been observed in the order-disorder separated quantum systems.[29–31]

Throughout our study, we have ignored the effect

of the electron-electron (e-e) correlation since the inclusion of the e-e correlation will not provide any new significant result in our present investigations.

Acknowledgment

I acknowledge with deep sense of gratitude the illuminating comments and suggestions I have received from Prof. S. Sil during the calculations.

References

[1] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).

[2] F. Hund, Ann. Phys. (Leipzig) 32, 102 (1938).

[3] M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A 96, 365 (1983).

[4] L. P. Levy, G. Dolan, J. Dunsmuir and H Bouchiat, Phys. Rev. Lett. 64, 2074 (1990).

[5] D. Mailly, C. Chapelier and A. Benoit, Phys. Rev. Lett. 70, 2020 (1993).

[6] V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher and A. Kleinsasser, Phys. Rev. Lett. 67, 3578 (1991).

[7] E. M. Q. Jariwala, P. Mohanty, M. B. Ketchen and R. A. Webb, Phys. Rev. Lett. 86, 1594 (2001).

[8] R. Deblock, R. Bel, B. Reulet, H. Bouchiat and D. Mailly, Phys. Rev. Lett. 89, 206803 (2002).

[9] M. Büttiker, Phys. Rev. B 32, 1846 (1985).

[10] H-F Cheung, E. K. Riedel and Y. Gefen, Phys. Rev. Lett. 62, 587 (1989).

[11] H. F. Cheung, Y. Gefen, E. K. Riedel and W. H. Shih, Phys. Rev. B 37, 6050 (1988).

[12] R. Landauer and M. Büttiker, Phys. Rev. Lett. 54, 2049 (1985).

[13] N. Byers and C. N. Yang, Phys. Rev. Lett. 7, 46 (1961).

[14] F. von Oppen and E. K. Riedel, Phys. Rev. Lett. 66, 84 (1991).

[15] I. O. Kulik, Physica B 284, 1880 (2000).

[16] K. Yakubo, Y. Avishai and D. Cohen, Phys. Rev. B 67, 125319 (2003).

[17] E. H. M. Ferreira, M. C. Nemes, M. D. Sampaio and H. A. Weidenmüller, Phys. Lett. A 333, 146 (2004).

[18] G. Montambaux, H. Bouchiat, D. Sigeti and R. Friesner, Phys. Rev. B 42, 7647 (1990).

[19] H. Bouchiat and G. Montambaux, J. Phys. (Paris) 50, 2695 (1989).

[20] B. L. Altshuler, Y. Gefen and Y. Imry, Phys. Rev. Lett. 66, 88 (1991).

[21] A. Schmid, Phys. Rev. Lett. 66, 80 (1991).

[22] M. Abraham and R. Berkovits, Phys. Rev. Lett. 70, 1509 (1993).

[23] A. Müller-Groeling and H. A. Weidenmuller, Phys. Rev. B 49, 4752 (1994).

[24] S. K. Maiti, Int. J. Mod. Phys. B 21, 179 (2007).

[25] S. K. Maiti, J. Chowdhury and S. N. Karmakar, Synthetic Metals 155, 430 (2005).

[26] S. K. Maiti, J. Chowdhury and S. N. Karmakar, J. Phys.: Condens. Matter 18, 5349 (2006).

[27] S. K. Maiti, Physica E 31, 117 (2006).

[28] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. 57, 287 (1985).

[29] Y. Cui, X. F. Duan, J. T. Hu and C. M. Lieber, J. Phys. Chem. B 104, 5213 (2000).

[30] J. Y. Yu, S. W. Chung and J. R. Heath, J. Phys. Chem. B 104, 11864 (2000).

[31] G. Zheng, W. Lu, S. Jin and C. M. Lieber, Adv. Mater. 16, 1890 (2004).

[32] J. X. Zhong and G. M. Stocks, Nano. Lett. 6, 128 (2006).

[33] J. X. Zhong and G. M. Stocks, Phys. Rev. B 75, 033410 (2007).

[34] C. Y. Yang, J. W. Ding and N. Xu, Physica B 394, 69 (2007).

[35] H. B. Chen and J. W. Ding, Physica B 403, 2015 (2008).

[36] W. Kohn, Phys. Rev. 133, A171 (1964).

[37] D. J. Scalapino, R. M. Fye, M. J. Martins, J. Wagner and W. Hanke, Phys. Rev. B 44, 6909 (1991).

[38] D. J. Scalapino, S. R. White and S. Zhang, Phys. Rev. B 47, 7995 (1993).

[39] G. Bouzerar, D. Poilblanc and G. Montambaux, Phys. Rev. B 49, 8258 (1994).